LOGIC EXERCISES: WEEK 2

QUESTION 1

What can you say about the formal validity of an argument that has:

i. premises consistent with the conclusion?
ii. premises inconsistent with the negation of the conclusion?
iii. premises inconsistent with one another?
iv. a logically true conclusion?
v. an inconsistent (i.e. logically false) conclusion?

QUESTION 2

Write out (full) truth-tables for these formulas. Which ones are logically true? Which ones are inconsistent (i.e. logically false)?

i. (P → Q)
ii. (P → P)
iii. (P → ¬Q)
iv. (¬Q → ¬P)
v. (P → (Q → P))
vi. ¬(P ∧ Q)
vii. (P → (Q ∧ R))
viii. ((P → R) ∨ (¬P → Q))
ix. (P ∨ (¬Q ↔ R))
x. ((P → R) → ((Q → R) → ((P ∨ Q) → R)))

QUESTION 3

Use (full) truth-tables to test the correctness of the following sequents. Give counterexample structures where appropriate.

i. (¬P → P) ⊨ P
ii. P ⊨ (Q → P)
iii. P, (P → Q) ⊨ Q
iv. (P → Q), ¬Q ⊨ ¬P
v. P, ¬P ⊨ Q
vi. (P → Q), (R ∨ ¬Q) ⊨ (Q → (P → R))
vii. (P → Q) ⊨ ((P → R) → (Q → R))
viii. ((P → Q) → Q) ⊨ (P ∨ Q)
ix. (P ∨ ¬ (Q → R)), (Q ↔ ¬(P ∧ R)), ¬R ⊨
x. ⊨ ((P ∨ (Q → R)) ↔ (R ∨ (Q → P)))

Formalise the following English claims as best as possible in L_1, noting any difficulties or potentially controversial points along the way.

- i. Although it was very hot, Her Majesty wore a cardigan.
- ii. Because it was very hot, Her Majesty stripped off.
- iii. Since there are no trees in Orkney, it is a desolate place.
- iv. Terry and Julie got married and had a baby.
- v. Unless I have made a mistake, the answer is 42.
- vi. Whether or not you like them, carrots are good for you.
- vii. If Oswald didn't shoot Kennedy, somebody else did.
- viii. If Oswald doesn't shoot Kennedy, somebody else will.
- ix. Even if Her Majesty was dying yesterday, she's perfectly healthy now.
- x. You can go to the bar now, but only if you have handed in your work.